Computational Linguistics I, Winter 2006. Marcus Kracht

To be submitted: Friday, February 17, 2006.

[A 4.1] Define a module of sets of strings and call it `StringSet`. Now define the following functions between string sets:

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\begin{aligned}
L \cdot M &:= \{\vec{x}^\frown\vec{y} : \vec{x} \in L, \vec{y} \in M\} \\
L/M &:= \{\vec{x} : \text{exists } \vec{y} \in M : \vec{x}^\frown\vec{y} \in L\} \\
L//M &:= \{\vec{x} : \text{forall } \vec{y} \in M : \vec{x}^\frown\vec{y} \in L\} \\
L\backslash M &:= \{\vec{x} : \text{exists } \vec{y} \in L : \vec{y}^\frown\vec{x} \in M\} \\
L\backslash\backslash M &:= \{\vec{x} : \text{forall } \vec{y} \in L : \vec{y}^\frown\vec{x} \in M\}
\end{aligned}
$$

[A 4.2] Calculate the strings of length at most 10 of the following expressions: $(\texttt{a}^*|\texttt{b}?)\texttt{ca}$, $\texttt{ab}^+|\texttt{ba}^+$, $(\texttt{aa}^*\texttt{b})^2$.

[A 4.3] Show that in general $L \cdot M = N$ iff (= if and only if) $L = N//M$ iff $M = L\backslash\backslash N$. *Hint.* This should follow directly from the definitions.

[A 4.4] Show that in general $(L^* \cdot M^*)^* = (L \cup M)^*$. *Hint.* One way to do this is as follows: Establish that (a) $(L^* \cdot M^*)^* \subseteq (L \cup M)^*$ and that (b) $(L \cup M)^* \subseteq (L^* \cdot M^*)^*$. Make use of the following principle: if $H \subseteq K^*$ then also $H^* \subseteq K^*$.