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Quantifiers: Semantics

E L Keenan, University of California, Los Angles, CA, USA

During the past 25 years, our empirical and mathematical knowledge of quantification in natural language has exploded. We now have mathematically precise (if sometimes contentious) answers to questions raised independently within generative grammar, and we are able to offer many new generalizations. We review these results here. For extensive overviews, see Westerståhl (1989), Keenan (1996), and Keenan and Westerståhl (1997). Some important collections of articles are van Benthem and ter Meulen (1985), Reuland and ter Meulen (1987), Gärdenfors (1987), Lappin (1988), van der Does and van Eijck (1996), Szabolcsi (1997), and from a mathematical perspective, Krynicki *et al.* (1995).

The best understood type of quantification in natural language is that exemplified by *all poets* in *All poets daydream.* We treat *daydream* as denoting a property of individuals, represented as a subset of the domain E of objects under discussion. Quantified Noun Phrases (NPs) such as *all poets* will be treated as denoting functions, called 'generalized quantifiers,' which map properties to truth values, True (T) or False (F). For example, writing denotations in boldface, **all poets** maps **daydream** to True (over a domain E) if and only if the set of poets is a subset of the set of objects that daydream. Interpreting Dets (Determiners) as functions from properties to generalized quantifiers, we give denotations for many quantifiers in simple set theoretical terms. We write $X \subseteq Y$ to say "X is a subset of Y," $X \cap Y$ for "X intersect Y," the set of objects that lie in both X and Y; $X - Y$ for the set of objects in X that are not in Y, and $|X|$ for the number of elements in X.

(1a) ALL(A) (B) = T iff $A \subseteq B$
(1b) (THE TEN) (A) (B) = T iff $|A| = 10$ and $A \subseteq B$
(1c) NO(A) (B) = T iff $|A \cap B| = 0$
(1d) SOME(A) (B) = T iff $|A \cap B| > 0$
(1e) NEITHER(A) (B) = T iff $|A| = 2$ & $|A \cap B| = 0$
(1f) (FEWER THAN TEN) (A) (B) = T iff $|A \cap B| < 10$
(1g) (ALL BUT ONE) (A) (B) = T iff $|A - B| = 1$
(1h) MOST(A) (B) = T iff $|A \cap B| > |A - B|$

To test that the definitions above have been properly understood, the reader should try to fill in appropriately the blanks in (2).

(2a) (AT LEAST TWO) (A) (B) = T iff ____
(2b) BOTH(A) (B) = T iff ____
(2c) (AT MOST FIVE OF THE TEN) (A) (B) = T iff ____
(2d) (LESS THAN HALF THE) (A) (B) = T iff ____

We concentrate on NPs of the form [Det + N], but we should point out three further classes of generalized quantifiers denoting NPs: first, lexical NPs, notably proper nouns such as *John* and *Mary*; second, boolean compounds in *and*, *or*, *neither* ... *nor* ... and *not*, as in *Neither John nor Mary* (came to the party), *Sue and some student* set up the chairs, *not more than two students* attended the lecture. And third, NPs built from Dets that combine with two Nouns to form an NP (Keenan and Moss, 1985; Beghelli, 1993): *more students than teachers* (signed the petition), *exactly as many students as teachers* signed, etc. The interpretation of *more* ... *than* ... is given by:

(3) For all properties A,B,C (MORE A THAN B) (C) = T iff $|A \cap C| > |B \cap C|$

The reader may want to write out the definitions of two place Dets such as FEWER ... THAN ..., TWO MORE ... THAN ..., TWICE AS MANY ... AS ..., THE SAME NUMBER OF ... AS ...

Standard Quantifiers: Some Linguistic Generalizations

We consider cases where the quantifier semantics sketched above has proven enlightening in formulating linguistic generalizations. As a first case, observe that in (4a) the presence of *ever*, called a 'negative polarity item (npi),' is ungrammatical; in (4b) it is fine.

(4a) *Some student here has ever been to Pinsk.
(4b) No student here has ever been to Pinsk.

Similarly *any* in *No child saw any birds on the walk* is an npi, not licensed in **Some child saw any birds on the walk*. **The linguistic query:** Which NPs in contexts like (4) license npi's? To within a good first approximation, the answer is given by the Ladusaw-Fauconnier Generalization (Ladusaw, 1983): *The NPs which license npi's are just those which denote decreasing (order reversing) generalized quantifiers.* A generalized quantifier F is *decreasing* iff whenever $A \subseteq B$ then if F(B) = T then F(A) = T. **No student** is decreasing since if all As are Bs ($A \subseteq B$), then if no student is a B, it follows that no student is an A (otherwise that student would also be a B, contrary to assumption). In contrast, **some student** is not decreasing: perhaps all As are Bs and there are many students among the Bs but they all lie among those that are not As. The reader can verify that the following NPs are decreasing and do license npi's: *fewer then five students*, *less than half the students*, *not more than two students*, *neither John nor Bill*, *no student's doctor.* In contrast, NPs such as *John*, *more than five students*, *most poets*, *either John or Bill*, *some student's doctor* are not decreasing and do not license npi's. Keenan and Szabolcsi (*see* **Boole and Algebraic Semantics**) point out that standard negation is also a decreasing function, and it also licenses npi's: **John has ever been to Pinsk* vs. *John hasn't ever been to Pinsk*.

Our first generalization also illustrates that whether an NP of the form [Det + N] is decreasing or not is decided by the Det. So if *no student* is decreasing then, so is *no child*, *no professional acrobat*, etc. since they all have the same Det. Similarly if *some student* is not decreasing, then neither is *some child*, *some professional acrobat*, etc., Many linguists refer to the expressions we call NPs as DPs ('Determiner Phrases'), in part because significant properties of the entire expression, such as whether it is decreasing or not, is decided by the choice of Det. (But other properties, such as whether the expression is animate, feminine, or satisfies the selection restrictions of a predicate, are determined by the N. #*Every idea laughed* is bizarre, since ideas aren't the kinds of things that can laugh and changing the Det does not improve matters: #*Some idea laughed*, #*Most ideas laugh*).

As a second generalization, consider how we may characterize the NPs that are *definite* (plural) in the sense that they may grammatically replace *the poems* in *two of the poems*. Some such NPs are *the ten poems*, *these (ten) cats*, *John's (ten) students*. Some NPs that are not definite (plural) in this sense are: *no poems*, *every cat*, *most students*. Again, to within a first approximation, we may say that the definite plural NPs are those of the form [Det + N], where (in each domain) Det denotes a function f that satisfies:

(5) For all A, either f(A) (B) = F; all B, or for some $X \subseteq A$, f(A) (B) = T iff $X \subseteq B$.

To illustrate the idea, **(John's ten)** (**students**) maps each set **B** to **F** if John does not have exactly ten students; if he does, then it maps a set B, such as **daydream in class** to **T** iff the property of being a student-which-John-has is a subset of B. See Barwise and Cooper (1981) and Matthewson (2001) for further discussion.

The two generalizations adduced so far are semantic characterizations of syntactic phenomena. For purposes of defining the class of well-formed expressions in English, we need to know which NPs license npi's in the predicate and which may occur naturally in the post *of* position in partitives. Most approaches to generative grammar desire a purely syntactic definition of these classes. Our observations do not rule out such definitions. Indeed, they provide a criterion for whether a proposed definition is adequate or not. Still, at the time of writing, we have no explicit syntactic definition of these two classes of NPs.

A third problem comparable to our first two is to characterize those NPs that occur naturally in *Existential There* (ET) contexts, as in (6).

(6a) There are/aren't *more than ten boys* in the room.
(6b) *There are/aren't *most boys* in the room.

This problem has vexed generative grammarians since Milsark (1977). See Reuland and ter Meulen (1987) and Keenan (2003). Part of the problem is that affirmative declarative Ss of the form in (6) have a variety of uses, with different uses seemingly allowing different NPs. For example, in so-called 'list contexts' (Rando and Napoli, 1978), ET sentences admit definite NPs such as *the bus* in *How can I get to UCLA from here? Well, there's always the bus, but it doesn't run very often*. But mostly these uses are not preserved in negative or interrogative Ss. Putting aside uses limited to affirmative declarative Ss, Keenan (2003) supports that NPs that occur freely in ET contexts are ones built from Dets that denote a certain kind of 'conservative' function. A Det is, standardly, **conservative**, or as we shall say here, **conservative on its first argument**, if it satisfies (7a), stated more generally in (7b).

(7a) Det poets daydream iff Det poets are poets who daydream
(7b) $D(A)(B) = D(A)(A \cap B)$

To see that *most but not all* is conservative, for example, check that *Most but not all poets daydream* and *Most but not all poets are poets who daydream* always have the same truth value. Indeed, the second sentence seems redundant, with the predicate just repeating information already contained in the Noun argument.

The formulation in (7b) says that in evaluating the truth of D(A) (B), we may limit the predicate argument B to those of its elements that occur in A. In this sense, then, we shall say that D is CONS1, 'conservative on its first argument.' And we support:

(8) **Cons** in general Dets denote CONS1 functions

Cons is a new generalization, not one that arises directly in response to a query from independent linguistic study. It is surprisingly strong. Given a domain E with n elements, Keenan and Stavi (1986) show that for $k = 4^n$ there are 2^k functions from pairs of properties to {T,F}. Only $2^{k'}$ of these functions are **Cons**, where $k' = 3^n$. So in a domain with just two individuals, there are $2^{16} = 65\,536$ maps from pairs of properties to truth values, only $2^9 = 512$ of which are conservative! So **Cons** rules out most logically possible denotations for Dets. Here is a simple non-conservative function, F: F(A) (B) = T iff |A| = |B|. Clearly F({a,b},{a,c}) = T, but F({a,b},{a,b} ∩ {a,c}) = F({a,b}{a}) = F, so F fails **Cons**.

Now, mathematically it makes sense to ask whether there are Dets that are conservative on their second, predicate, argument. Such Dets would denote functions D satisfying D(A) (B) = D(A ∩ B) (B), where we can limit the As we consider to those that lie in B. Many natural classes of Dets fail CONS2. For example, **universal** Dets, such as *all*, *all but ten*, *every . . . but John*. It might be false that all poets daydream, but it must be true that all poets who daydream daydream. So *all* is not CONS2. Also not CONS2 are **definite** Dets, such as *the*, *the ten*, *these (ten)*, *John's (ten)*; and **proportional** Dets, such as *most*, *half (of) the*, and *not one . . . in ten*. But there is one large class of Dets that are CONS2. They include the *intersective* Dets, ones whose values just depend on which objects have both the Noun property and the Predicate property. For example, *some* is intersective since the truth of SOME(A) (B) is decided just by checking A ∩ B (verifying that it is not empty). Let us define:

(9) D is *intersective* iff for all A, A′, B, B′ if A ∩ B = A′ ∩ B′ then D(A) (B) = D(A′) (B′)

So intersective Ds are ones 'invariant' under replacement of A and B with other arguments A′ and B′ provided the intersection of the pairs of arguments remains unchanged. Note that, an intersective D is necessarily CONS 1: since A ∩ B = A ∩ (A ∩ B) we infer D(A) (B) = D(A) (A ∩ B). And since A ∩ B = (A ∩ B) ∩ B, we have that D(A)(B) = D(A ∩ B)(B), and so intersective Dets are CONS2. In fact:

Theorem D is intersective iff D is CONS1 and CONS2 (Keenan, 2003)

An important special case of intersective Dets are **cardinal** ones, whose values depend on the cardinality of A ∩ B, such as *at least n*, *more/fewer than n*, *at most n*, *approximately n*, *between n and m*, *several*, *a dozen*, *just finitely many*, and *infinitely many*. And we now answer our third query (Keenan, 2003):

(10) NPs which occur freely in Existential There contexts are (boolean compounds of) ones built from CONS2 Dets.

'Boolean compounds' here just refers to NPs built by conjunction, disjunction, and negation. For example, *There are at least two dogs and not more than five cats in the garden* is fine since *at least two dogs* and *more than five cats* both occur in ET contexts.

Example (10) predicts that NPs built from intersective Dets occur in ET contexts, and this is correct: *Aren't there between five and ten students in your class? Was there no student (but John) in the building?* etc. (Note that **no . . . but John** treated as a Det is intersective, as *No A but John is a B* is True iff A ∩ B = {John}, that is, the only A that is a B is John). Similarly, the universal, definite, and proportional Dets noted earlier are predicted not to occur freely in ET contexts since they fail CONS2, and this is correct: * *Weren't there all/most/the students in the class?*

Are there are CONS2 Dets that fail to be intersective? By the theorem, they would have to fail to be CONS1, and so rather rare. But there are two candidates: NPs of the form [only/mostly N], such as *only poets*. Interpreting *only* as a Det yields ONLY(A) (B) = T iff B ⊆ A. For example, *Only poets daydream* is true iff everyone who daydreams is a poet (but there may be poets who don't daydream). Clearly ONLY thus defined is CONS2, since ONLY(A ∩ B)(B) = T iff B ⊆ A ∩ B, iff B ⊆ A. So (10) predicts the well-formedness of *There weren't only poets at the party*, which is correct.

A last case covered, unexpectedly, by (10) are NPs built from comparative Dets like *more . . . than . . .* which combined with two Ns to form an NP. Each N property is a conservativity domain. To evaluate whether *More students than teachers daydream* we must consider both the students who daydream and the teachers who daydream. But not only are comparative NPs conservative on their two N arguments; they are intersective, in fact, cardinal. To decide whether more As than Bs have C, we need only check |A ∩ C| and |B ∩ C|, verifying that the former is greater than the latter. Thus, (10) predicts that cardinal comparatives should occur in ET contexts, and they do: *Weren't there more students than teachers at the party?* (We can also compare predicate properties, as in *More poets drink than smoke*, in which case only the single N property is a conservativity domain).

A fourth linguistic generalization is given in (11), where 'lexical' means 'not syntactically derived.'

(11) Lexical NPs are monotonic, and lexical Dets build monotonic NPs.

A monotonic NP is one that either denotes a decreasing function (already defined) or an increasing one, where a generalized quantifier F is **increasing** iff whenever $A \subseteq B$ then if $F(A) = T$ then $F(B) = T$. Lexical NPs, principally proper nouns (*John*,...) and pronouns (*he*, *she*,...), are easily seen to be increasing: to paraphrase Aristotle, if all poets daydream and Paul is a poet then Paul daydreams. The reader may verify that *all/some/most/the five/my/John's cats* are all increasing, so lexical Dets usually build increasing NPs. But *no* and *neither* build decreasing ones. The only slightly doubtful case are NPs built from bare numerals, such as *two poets*. The NP is increasing if *two* is understood in the sense of *at least two*; it is not monotonic if understood as *exactly two*. We find the *at least two* reading the most natural because in some cases that is clearly what is intended, as in *Are there two free seats in the front row?* Additional information from context can be invoked to force the *exactly two* reading. Our last generalization in this section is (12):

(12) Natural Language Dets are Domain Independent

In practical applications, such as in database theory (Abiteboul *et al.*, 1995), it is important to know what the domain of objects is that properties under consideration are subsets of. It would be more accurate (van Benthem, 1984) to treat NPs, for example, as functions that associate with each possible domain E a generalized quantifier over E, a function associating with each subset A of E a truth value. Similarly Dets associate with each E a function mapping subsets of E to generalized quantifiers over E. But (12) says that the value a Det denotation D assigns to a pair A, B of properties cannot depend on the choice of underlying domain (as long as A and B are subsets of that domain). This (seemingly vacuous) constraint entails, for example, that natural languages could not present a Det *blik* defined by: *Blik As are Bs* is True iff the number of non-As is two. The truth value of such an S would vary with the domain: if $A = \{a\}$ and $E = \{a,b,c\}$ then it is True, but if $E = \{a,b\}$ it is False.

To close this section, we note that our treatment of NPs and Dets enables us to specify precisely certain traditional, if informally given, classes of expressions. We note two cases. First, the linguistic literature on quantification usually builds heavily on the specific quantifiers *some* and *all*, which we have already defined. But now we can see that they represent two quite general classes of Dets: the *Existential* ones, including *some*, are just those that denote intersective functions, as defined in (9). The *Universal* Dets are those that denote cointersective ones, as defined in (13).

(13) D is *co-intersective* iff for all A, B, X, Y if $A - B = X - Y$ then $D(A)(B) = D(X)(Y)$

Thus, whether a cointersective function D is True of a pair A,B is decided just by checking $A - B$. To see that ALL is cointersective observe that, as defined, ALL(A) (B) = T iff $|A - B| = 0$. (ALL BUT TEN) (A) (B) = T iff $|A - B| = 10$, etc. Our approach also enables us to see that proportionality Dets such as *most*, *half*, *more than one ... in ten*, etc., are more complicated than either intersective or cointersective Dets, because their truth at a pair A, B of properties depends on both $A \cap B$ and $A - B$.

Secondly, most of the Dets discussed in the linguistic literature are ones with a 'logical' or 'mathematical' sense: *some*, *every*, *most*, *most of the ten*, *not all*, *most but not all*, *between a third and two thirds of the*, etc. But we have also countenanced a few which are more 'empirical', such as *my*, *John's ten*, *no... but John*, etc. (see Keenan, 1996). Can we say in precise terms what the distinction is? We can. The logical' Dets are those that are invariant under permutations of the elements of the underlying domain E. A permutation of E is simply a one-to-one function from E onto E. If h is such a function and A a subset of E, then by h(A) is meant $\{h(x)|x \text{ in } A\}$. And we say that a generalized quantifier F is *permutation invariant* (PI) iff for all A, all permutations h of E, $F(A) = F(h(A))$. A Det function D is PI iff for all A,B and all permutations h, $D(A)(B) = D(h(A))(h(B))$. Note that, for any $X \subseteq E$, X and h(X) always have the same cardinality. So the 'logical' quantifiers are those that cannot distinguish between properties A, A′ of the same cardinality (and whose complements, $E - A$ and $E - A'$ have the same cardinality when E is infinite). One computes, then, that Dets such as *some*, *every*, *most*, *most of the ten*, *not all*, *most but not all*, *between a third and two thirds of the* always denote PI functions, whereas ones like *my*, *John's ten*, etc., do not.

Some Non-Standard Quantifiers

We begin by considering the interpretation of the quantified NPs already discussed when they occur as objects of transitive and ditransitive verbs: *John envies all movie actors*, *He gave most of his teachers several presents*. Many semanticists regard these uses as illustrating a 'type mismatch.' To interpret the object NP in *John interviewed every applicant* we apparently need to treat it as a function mapping a binary relation (**interview**) to a property, but we are already committed to interpreting NPs as maps from properties (unary relations) to truth values (zero-ary relations). But in fact there is no problem here at all. We know exactly what property *interviewed every applicant* denotes. It is the set of objects x such that 'x interviewed every applicant', that is, such that **every applicant** is true of the *set* of objects that x interviewed. In the last phrase, we are applying the generalized quantifier **every applicant** to a set – which is exactly how we have already been interpreting NPs. So the value that an NP denotation assigns to a binary (ternary, . . .) relation is determined by the value that it assigns to sets (properties). So the solution to the 'type mismatch' problem is to treat NPs as directly denoting functions mapping n+1-ary relations to n-ary ones in such a way that their values at n > 1-ary relations are determined by their values at the unary relations. Here is the solution for binary relations. Keenan and Westerståhl (1997) give the general statement. We write aR for {b|aRb}, the set of objects that a stands in the relation R to.

(14) A type <1> function F over a universe E maps each subset of E to a truth value and each binary relation R over E to a subset of E by: $F(R) = \{a|F(aR) = T\}$.

Clearly, then, each generalized quantifier uniquely determines a type <1> function; all we have done is add more objects (binary relations) to its domain. So the interpretation of an S with two quantified NPs is given compositionally, as in (15).

(15a) No politician kissed every baby
(15b) (**no politician**) ((**every baby**) (**kissed**))

Note that, (15b) means 'No politician has the property that he kissed every baby,' the object narrow scope (ONS) reading of (15a). In fact, (15a) does not have an object wide scope (OWS) reading, on which it would mean that every baby has the property that no politician kissed him. But some Ss with two quantified NPs do present such ambiguities, a matter of much concern to linguists (Szabolcsi, 1997). *Some editor read every manuscript* has an ONS reading, representable analogously to (15b), on which it means that there is an editor with the property that he read every ms. But it also has a OWS reading, on which it means that every ms has the property that some editor read it – so the editors may vary with the manuscripts. An easy use of variable binding operators (VBOs) allows us to represent the less accessible OWS reading by:

(16) (**every manuscript** x) ((**some editor**) (**read** x))

The use of VBOs has distracted us from the fact that the range of logically possible interpretations of nonsubject NPs is vastly greater than of subject NPs. Given a universe E with more than one element, there are many more functions from binary relations to properties than there are from properties to truth values. And natural languages provide the means for denoting some of these. To see this, we need a way to test functions H from relations to properties to see if they are possibly extensions of functions from properties to truth values. Here is such a test, given first by example, where we test X:

(17a) If John praised exactly the people who Bill criticized then John praised X iff Bill criticized X
(17b) If $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$

For example, *most of the Peter's students* passes the X test in (17a). Given the truth of the *if*-clause, we infer that *John praised most of Peter's students* and *Bill criticized most of Peter's students* have the same truth value. (This is hardly surprising; the NP we are testing occurs as a subject of a P1 and thus is interpretable as a generalized quantifier.)

But consider now the reflexive pronoun *himself*. It fails the test. Imagine, for example, that the *if*-clause is true and that John praised just Sam, Frank, Bill, and Sue. Then those are just the people that Bill criticized, so *Bill criticized himself* is true, but *John criticized himself* is false. Hence, there is *no* function from properties to truth value that takes exactly the value on binary relations as **himself** does. That is, reflexives represent an increase in logical expressive power. Moreover *himself* is not unique here; all (nontrivial) referentially dependent NPs in object position are logically new in this sense. This includes ones like *everyone but himself*, *both himself and the teacher*, *everyone smarter than himself* (as in *John criticizes everyone smarter than himself*) in which the NP must be referentially dependent, as well as NPs like *his mother* where it simply may be dependent. It is the dependent interpretation where it is new.

The increase in logical expressive power afforded by nonsubject NPs is far more extensive than instantiated by the referentially dependent NPs. They are not functions of type <1>, but they can be correctly interpreted as functions from binary relations to sets, (just not ones that extend appropriately a generalized quantifier). But the sort of dependency in (18) cannot be handled in a comparable way:

(18) Different people like different things

The weakest truth conditions of (18) are easy to state: for any two (different) people x and y, the set of things that x likes is not identical to the set that y likes. And we know (Keenan, 1996) that over any domain with several people and things, there are *no* type <1> functions F and G such that for all binary relations R, F(G(R)) is true iff different people stand in the relation R to different things. Similar claims hold for other Ss which involve comparing different object sets with different choices of subject argument: *John and Bill support rival political parties*, *Rosa and Zelda date men who dislike each other*, etc. In fact, the same claim holds even when the comparison is not one of difference: *All the students answered the same questions on the exam*, *They wore the same color necktie*, etc. Speaking very informally, we can say that the combination of subject and object expressions in these Ss place conditions on the relation denoted by the transitive verb which are inherently 'relational,' not expressible as independently statable conditions on each argument. To state this more explicitly, let us define:

(19) A function H from binary relations to truth values is said to be of type <2>. Such a function is type <1> *reducible* iff there there are type <1> functions F,G such that for all binary relations R, H(R) = F(G(R)).

Then what we are saying above is that the type <2> functions expressed by (Different people, different things), (John and Bill, rival political parties), etc., are not type <1> reducible. Thus, they are not expressible as the composition of two generalized quantifiers, and so are logically new. Some additional examples are induced by the *else-else* construction in (20), the *which-which* construction in (21), reciprocal objects in (22), and predicate anaphors in (23). For further examples, see Keenan (1996).

(20) John didn't praise Bill but everyone else praised everyone else.
John praised Bill but no one else praised anyone else.

(21) John doesn't know which students answered which questions on the exam.

(22) The students were shouting at each other.

(23) John read more books than Bill (did).

In sum, we have a list of expression types that are not type <1> reducible, but we do not know precisely what type <2> functions are expressible. All of them (over finite E)?

Thus, much remains to be discovered even in English, the language in which quantifiers have been the most extensively studied. And we are really just beginning to study quantification in less well known languages (Bach *et al.*, 1995; Matthewson, 2001) as well as in contexts other than that of NPs and Dets, the most prominent here being temporal and event quantification, using adverbial quantifiers, as in *Matt always/often/occasionally/seldom/rarely/never visits museums on weekends* (de Swart, 1996).

Answers to Exercises
(AT LEAST TWO) (A) (B) = T iff $|A \cap B| \geq 2$
BOTH(A) (B) = T iff $|A| = 2$ and $A \subseteq B$
(AT MOST FIVE OF THE TEN) (A) (B) = T iff $|A| = 10$ and $|A \cap B| \leq 5$
(LESS THAN HALF THE) (A) (B) = T iff $|A \cap B| < |A|/2$

See also: Boole and Algebraic Semantics; Formal Semantics; Monotonicity and Generalized Quantifiers.

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Quantity

B Hermans, Tilburg University, Tilburg, The Netherlands

In many languages, quantity has a contrastive function. In languages of this type, certain segments behave phonologically as if they really are two segments. These segments contrast with qualitatively identical segments that behave phonologically as single segments. An example of a language with a quantity contrast among vowels, as well as consonants is Luganda (Ganda). This is illustrated by the following words, taken from Katamba (1989).

(1) phonological form	phonetic realization	
/kula/	[kula]	'grow up, IMP.'
/kuula/	[ku:la]	'uproot, IMP.'
/ta/	[ta]	'release, IMP.'
/tta/	[t:a]	'kill, IMP.'

In the majority of these languages, the phonological contrast between double (geminate) segments and single segments is phonetically implemented as a contrast between segments with greater duration versus segments with shorter duration. One of the languages where this is the case is Luganda, as shown by the examples above. This should not be taken to mean that in these languages a geminate segment is always phonetically longer than a corresponding single segment by a fixed amount of time, since there are many factors that influence phonetic duration. Some of the more important phonetic factors are the following: all else being equal, lower vowels are longer than higher vowels (Lehiste, 1970); vowels in open syllables are longer than vowels in closed syllables (Maddieson, 1985); vowels preceding voiced consonants are longer than those preceding voiceless consonants (Chen, 1970); vowels preceding fricatives are longer than vowels preceding stops (House, 1961); stressed vowels are longer than unstressed ones (Beckman *et al.*, 1992). Given these complicating factors, it can happen that a phonological geminate is realized with only little more phonetic length, or even less phonetic length, than a phonologically simple segment.

In almost all cases, the quantity contrast is binary. Apparent examples of ternary contrasts can almost always be reduced to a binary contrast by one of the phonetic tendencies listed above. Thus, in Estonian the third consonantal quantity is really a phonologically geminate consonant that is phonetically lengthened in a stressed syllable (Lehiste, 1966; Prince, 1980). Another, particularly convincing case is KiKamba (Kamba). Roberts-Kohno (1995) shows that in this language, the multiple quantity degrees attested on vowels can be reduced to a binary contrast by a system of phonetic lengthening rules. Yet apparently not all multiple quantity degrees can be reduced to a binary contrast. Ladefoged and Maddieson (1996) agree with Hoogshagen (1959) that Mixe is a language with a ternary quantity contrast on vowels.

In the classical generative literature (Chomsky and Halle, 1968), quantity was represented with the binary feature [long]. It soon became clear that this representation is wrong because it cannot capture the most fundamental property of geminate sounds, which is that they behave as a sequence of two single segments with respect to many phonological phenomena. This general phenomenon can take many specific forms. Some of the most frequent