

## MATHEMATICAL LINGUISTICS II (Ling 218)

Topic: Mathematical results on quantification in natural language  
(and the boolean bedrock on which they stand)

Instructor: Edward L. Keenan

Meeting time and place: Winter quarter 2009  
Tuesday from 1:00 – 4:00  
Haines Hall A82

Prerequisites:

Familiarity manipulating set theoretical operations  
Elementary exposure to model theory and boolean algebra

Readings:

Boolean structure

Required one class handout (electronic). It draws from

Background

1. Chs 3 and 5 in E. Mendelson *Boolean Algebra and Switching Circuits* (Schaum's Outline Series) 1970. McGraw-Hill
2. Chs 1 and 2 in volume 1 of *Handbook of Boolean Algebras*. 1989 Sabine Koppelberg. (eds) J. Donald Monk with Robert Bonnet. North-Holland .
3. Part 1 of *Boolean Semantics for Natural Language* E.L. Keenan and L.M. Faltz. D. Reidel Pub. Co. 1985.

Quantificational structure:

Required (electronic)

E.L.Keenan "Quantifiers" to appear in *Handbook of Semantics* (2<sup>nd</sup> edition)

K. von Heusinger, C. Maienborn, & P. Portner editors. Mouton de Gruyter.

E.L. Keenan and D. Westerståhl. 1997. "Generalized quantifiers in linguistics and logic" Pp. 837 – 895 in *Handbook of Logic & Language* J. van Benthem and A.ter Meulen (editors). North-Holland.

Background (strongly recommended as a textbook for this course)

S. Peters and D. Westerståhl. 2006. *Quantifiers in Language and Logic*. OUP

Supplemental suggested on a topic by topic basis as the course proceeds.

### Lecture Outline (Subject to Expansion / Contraction)

Weeks 1 – 3

Classification of English quantifiers of (Lindström) type (1,1) (= ((e,t),((e,t),t)).

Intersective (= Generalized Existential), noted INT. Cardinal  $\subset$  INT.

Co-Intersective (Generalized Universal), noted CO-INT. Co-cardinal  $\subset$  CO-INT.

Proportionality Quantifiers (*most, seven out of ten, less than half (the),...*)

General properties: *Conservativity* (CONS) and *Universe Independence* (van Benthem) characterize English (1,1) quantifiers – a strong claim, as many complex expressions are of type (1,1). *Automorphism Invariance* and “logicality”; basic isomorphism and closure theorems e.g.  $INT \cong CO-INT \cong \text{Type } (1) = ((e,t),t)$ , the generalized quantifiers. the boolean closure of  $INT \cup CO-INT = CONS$

*Query* Why are natural language quantifiers generally conservative? Why do we expect type (1) functions in English? *Speculative Answer* (swallow hard): boolean operations are, as Boole thought, properties of mind (innate). We need individuals for direct reference, and Type (1) is (we prove) the boolean closure of the set of individuals. INT and CO-INT do not increase logical expressive power as they are isomorphic to Type (1). And CONS is the boolean closure of  $INT \cup CO-INT$ .

NB: Type (1) is really type (n+1,n) – they map n+1-ary relations to n-ary ones.

#### Week 4

Mass terms and abstract terms: are they modeled by atomless boolean lattices?  
The Lindenbaum-Tarski algebra for Sentential Logic: proof of atomlessness and incompleteness. Vaught’s isomorphism theorem (hopefully). A suggestion for a novel approach to intensional logic.

#### Week 5

Beyond the Frege Boundary; Sortal Reducibility  
English quantifiers of high(er) type:  $((1,1),1)$ ;  $(1,(1,1))$ : maybe  $((1,1),(1,1))$   
*More students than teachers surf the net; More guests came early than left late;*  
*?More students came early than teachers left late*  
Resumptive quantification: *Most men are taller than most women*  
Quantifiers of type (2):  
*Different people like different things*  
*John danced with Mary but no one else danced with anyone else*  
A puzzlette: *Rosa and Zelda date men who dislike each other.* Its primary reading is non-Fregean. How to get it? Replace *date* with *attract* we lose it. Why?

#### Week 6

Some specifically linguistic elements of type (1), (1,1) and  $((1,1),1)$   
Possessives: *every student’s but no teacher’s bicycle was stolen*  
Exception Phrases: *Every/No student but John signed the petition*  
Negative Polarity Item Licensors:  
Less than half the students here have ever seen a baobob tree  
\*?More than half the students here have ever seen a baobob tree  
Definites Two of John’s / the / those students got a perfect score on the exam  
\*Two of no teacher’s / most / between five and ten students got ...  
More of Mary’s than of Susan’s papers were accepted

Week 7

Argument Anaphors: type (2,1) and more generally (n+2,n+1).

*Each student praised everyone but himself*

Predicate Anaphors: *Rosa read more plays than Zelda*

*More speculation: Why do natural languages present scope ambiguities? Ans: Those are the ways that maps of type (1,0) extend faithfully to ones of type (2,1) per the Argument Structure Theorem*

Week 8

First Order Definability: Proofs of non-first order definability of proportionality quantifiers as well as comparative cardinal quantifiers (*more...than...*, etc.).

Applications of Ehrenfeucht-Fraïssé games (hopefully)

Weeks 9 and 10: Open

Maybe some thoughts on temporal quantifiers:

*John never, rarely, occasionally, usually, often, always jogs to school*

Some relations between quantifiers and adjectives.