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Duality

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One of the key ideas of modern mathematics and linguistics is the notion of invariance under permutation. On the mathematical side, Kaplansky writes:

> We take this opportunity to mention Klein's *Erlangen* program. In a famous lecture, Felix Klein enunciated the thesis that the goal of geometry is the study of properties invariant under a particular group of transformations, the group being selected appropriately for the geometry in question.
>
> Klein's program is widely quoted, and of considerable historical significance. But it might be a good idea to point out that geometry is not unique in possessing an *Erlangen* program. In the study of any mathematical system, the group of automorphisms is certain to be important.... (Kaplansky 2003:p. 86)

Keenan and Stabler (2003) explore extensively the linguistic applicability of the insight Kaplansky expresses in the last sentence of the quotation above.

Another key idea of mathematics is the notion of duality. In what follows, I attempt to show, through some initial examples, that duality yields linguistic insights as well. I hope to pursue further examples subsequently.

1 Mathematics

In mathematics, duality arises naturally in a variety of contexts.

Set Theory. Halmos (1960) points out:

> ...the theorems of set theory usually come in pairs. If an inclusion or equation involving unions, intersections, and complements of subsets of E we replace each set by its complement, interchange unions and intersections, and reverse all inclusions, the result is another theorem. This fact is sometimes referred to as the *principle of duality* for sets. [p. 18]

Lattice Theory. Birkhoff (1967):

> ...It is obvious from inspection of conditions P1-P3 [the reflexivity, transitivity, and anti-symmetry conditions defining posets] that

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THEOREM 2 (DUALITY PRINCIPLE). *The converse of any partial ordering is itself a partial ordering.*

DEFINITION. The *dual* of a poset X is that poset $\check{X}$ defined by the converse partial ordering relation on the same elements.[p. 3]

Lattice Theory again. Davey and Priestley (1990) elaborate:

1.17 The dual of an ordered set. Given any ordered set P we can form a new ordered set P^∂ (the **dual** of P) by defining $x \leq y$ to hold in P^∂ if and only if $y \leq x$ holds in P. For P finite, we obtain a diagram for P^∂ simply by 'turning upside down' a diagram for P. ...

To each statement about P there corresponds a statement about P^∂. ... In general, given any statement Φ about ordered sets, we obtain the **dual statement** Φ^∂ by replacing each occurrence of $\leq$ by $\geq$ and vice versa. Thus ordered set concepts and results hunt in pairs. This fact can often be used to give two theorems for the price of one or to reduce work .... The formal basis for this observation is the Duality Principle below; its proof is a triviality.

1.18 The Duality Principle. *Given a statement Φ about ordered sets which is true in all ordered sets, then the dual statement Φ^∂ is true in all ordered sets.*[pp. 12f.]

Category Theory. Mac Lane (1971) develops the concept of duality in Category Theory as follows:

The *dual* of any statement Σ of [the elementary theory of an abstract category] is formed by making the following replacements throughout in Σ: "domain" by "codomain", "codomain" by "domain" and "h is the composite of g with f" by "h is the composite of f with g"; arrows and composites are reversed. Logic (and, or, ...) is unchanged. This gives the following table ....

Statement Σ	Dual statement Σ^*
$f : a \to b$	$f : b \to a$
$a = \mathrm{dom} f$	$a = \mathrm{cod} f$
$i = 1_a$	$i = 1_a$
f is monic	f is epi
u is a right inverse of h	u is a left inverse of h
f is invertible	f is invertible
t is a terminal object	t is an initial object

Note that the dual of the dual is the original statement ($\Sigma^{**} = \Sigma$). If a statement involves a diagram, the dual statement involves that diagram with all the arrows reversed. [pp. 31f.]

Category Theory again. Barr and Wells (1990) define the dual of a category as follows:

2.6.7 The dual of a category. Given any category $\mathscr{C}$, you can construct another category denoted $\mathscr{C}^{\mathrm{op}}$ by reversing all the arrows. The **dual** or **opposite** $\mathscr{C}^{\mathrm{op}}$ of a category $\mathscr{C}$ is defined by:

D-1 The objects and arrows of $\mathscr{C}^{\mathrm{op}}$ are the objects and arrows of *calC*.
D-2 If $f : A \to B$ in $\mathscr{C}$, then $f : B \to A$ in $\mathscr{C}^{\mathrm{op}}$.
D-3 If $h = g \circ f$ in $\mathscr{C}$, then $h = f \circ g$ in $\mathscr{C}^{\mathrm{op}}$.

The meaning of D-2 is that source and target have been reversed. It is easy to see that the identity arrows have to be the same in the two categories $\mathscr{C}$ and $\mathscr{C}^{\mathrm{op}}$ and that [the statements defining a category] hold, so that $\mathscr{C}^{\mathrm{op}}$ is a category.
...

...The major use we make of the concept of dual is that many of the definitions we make have another meaning when applied to the dual of a category that is often of independent interest. [pp. 33f.]

The category-theoretic characterizations generalize the characterizations from sets and lattices: a preorder—that is, a set on which there is defined a relation that is reflexive and transitive (but not necessarily anti-symmetric)—may be regarded as a category whose objects are the elements of the set and whose arrows correspond to the partial ordering relation. In particular, identity arrows correspond to the reflexivity requirements and transitivity corresponds to composition. Reversing these arrows gives the dual preorder. But the category-theoretic definition generalizes the set-theoretic and lattice-theoretic perspectives. The linguistic examples that follow are not formalized categorically, ...at least not yet.

2 Words ↔ Worlds

2.1 Anscombe

In the mid-1950's, Elizabeth Anscombe (1957) offered an essential insight in her account of the different roles a list might play with respect to a shopping expedition:

> 32. Let us consider a man going round a town with a shopping list in his hand. Now it is clear that the relation of this list to the things he actually buys is one and the same whether his wife gave him the list or it is his own list; and that there is a different relation when a list is made by a detective following him about. If he made the list itself, it was an expression of intention; if his wife gave it him, it has the role of an order. What then is the identical relation to what happens, in the order and the intention, which is not shared by the record? It is precisely this: if the list and the things that the man actually buys do not agree, and if this and this alone constitutes a *mistake*, then the mistake is not in the list but in the man's performance (if his wife were to say: 'Look, it says butter and you have bought margarine', he would hardly reply: 'What a mistake! we must put that right' and alter the word on the list to 'margarine'); whereas if the detective's record and what the man actually buys do not agree, then the mistake is in the record.

The relation of the two lists to events contains a common core: a correspondence between the items on the list and the purchases. And this correspondence is the same. That is, whether our shopper wants to pair a particular pound of butter with the list item 'butter' or our detective wants to pair up the word 'butter' with that same pound of butter, the criteria for the match or pairing are the same. The asymmetry in the two cases that Anscombe points out stems not from the correspondence itself but from whether we go from list to object or go from object to list.

We regard this as an instance of duality: a single relation—the matching relation between list and purchases—can be viewed from two different perspectives.

2.2 Searle

Two decades later, John Searle proposed a classification of illocutionary acts in which Anscombe's insight plays a key role. The classification rests on 'twelve significant dimensions of variation'. The first of these is the point or purpose of the act.[1] The second is what Searle calls 'direction of fit':

> Some illocutions have as part of their illocutionary point to get the words (more strictly, their propositional content) to match the world, others to get the world to match the words. Assertions are in the former cateogry, promises and requests are in the latter. The best illustration of this distinction I know of is provided by Elizabeth Anscombe (1957). [see above] ...
>
> In these examples the list provides the propositional content of the illocution and the illocutionary force determines how that context is supposed to relate to the world. I propose to call this difference a difference in *direction of fit*. The detective's list has the *word-to-world* direction of fit (as do statements, descriptions, assertions, and explanations); the shopper's list has the *world-to-words* direction of fit (as do requests, commands, vows, promises). I represent the word-to-world direction of fit with a downward arrow thus $\downarrow$ and the world-to-words direction of fit with an upward arrow thus $\uparrow$. Direction of fit is always a consequence of illocutionary point. It would be very elegant if we could build our taxonomy entirely around this distinction in direction of fit, but though it will figure largely in our taxonomy, I am unable to make it the entire basis of the distinctions.

Searle then proposes a 5-fold taxonomy of illocutionary acts (after a critical review of Austin's classification in *How to Do Things with Words* (Austin (1970))), based primarily on the parameters: illocutionary point, direction of fit, psychological state, and propositional content. He symbolizes these as follows:[2]

[1] Examples: the point or purpose of an order is to get someone to do something, the point or purpose of an assertion is to represent something, the point of puporse of a promise is to undertake an obligation. Each of these examples is an instance of a larger family.

[2] We are primarily interested in the direction of fit properties of this taxonomy, but there are a lot of extra symbols. In the illocutionary point column, '$\vdash$' is Frege's assertion sign, '!' is commonly used to indicate commands; in the psychological state column, B stands for a family of states related to believing, W stands for a

taxon	*illocutionary point*	*direction of fit*	*psychological state propositional content*
assertives	$\vdash$	$\downarrow$	*B(p)*
examples: *believing that p, asserting that p, insisting that p, suggesting that p, ...*			
directives	!	$\uparrow$	*W(H does A)*
examples: *asking H to A, ordering H to A, requesting H to A, begging H to A, ...*			
commissives	C	$\uparrow$	*I (S does A)*
examples: *promising to A, vowing to A, pledging to A, contracting to A, ...*			
expressives	E	$\emptyset$	*(P)(S/H + property)*
examples: *apologizing, thanking, congratulating, welcoming, deploring, ...*			
declarations	D	$\updownarrow$	$\emptyset$ *(p)*
examples: *nominating, appointing, ...*			
(assertive declarations)	D_a	$\downarrow\updownarrow$ *B(p)*	
examples: calls by umpires and referees, jury findings, ...			

Anyone familiar with distinctive feature theory is bound to feel that columns 2, 3, and 4 are hardly exhaustive of the primitives proposed—this is a sparse classification. Moreover, it doesn't assign any role to iterative speech acts (such as hectoring a witness) or to second order speech acts (insulting, libeling, ...). And of course it would be wonderful to know what C.S. Peirce would make of it.[3] We aren't interested here in a theory of speech acts in general (at least, not initially). But we are interested in Searle's use of the direction of fit.

For Anscombe, there are two 'directions of fit': from list to purchases and from purchases to list. Searle expands on this to include the class he calls declarations, of which he states:

> *Declarations.* It is the defining characteristic of this class that the successful performance of one of its members brings about the correspondence between the propositional content and reality, successful performance guarantees that the propositional concern [content?] corresponds to the world: if I successfully perform the act of appointing you chairman, then you are the chairman;.... [p. 149]

Talk about dynamic semantics! (*Fiat lux!* was, thank God, just the beginning.)

Searle distinguishes two subclasses of declarations—those that express a ruling or decision concerning the institutional status of independently existing states of affairs (judicial findings, referee decisions, ...) and whose successful performance brings about the existence of an institutional fact of the appropriate kind, and those that are not necessarily institutionally dependent, though they may be (naming, defining, abbreviating, ...).[4]

family of states related to wanting or wishing, *I* stands for a family of states related to intending, *P* is a variable ranging over the relevant range of psychological states, *S* is speaker, *H* is hearer, *A* is action, *p* is proposition, and C, E, D represent the illocutionary point of Commissives, Expressives, and Declarations, respectively.

[3] To explain a bit, if we think about the information carried by speech (and other signaling channels), Peirce's early *icon/index/symbol* trichotomy is still useful: speech acts typically involve properties of each.

[4] As with many other cases, the intuitive difference Searle is sensitive to here involves a distinction among acts, not necessarily a distinction among verbs.

2.3 Preliminary Assessment

Anscombe's insight is both appealing and correct—correct in the sense that the matching principle between word and world is the same in the two instances she highlights. The asymmetry between the two cases is a temporal one: either the list precedes the purchases or the purchases precede the list. This doesn't match up exactly with English usage: assertives and directives have very different aux/modal properties, for example. An additional problem involves the interaction, in Searle's notation, of the direction of fit with psychological operators: in the schema for commissives, for example, we have $C \uparrow$ (*I(S does A)*, but this doesn't sort out (formally, at least) what the scope of the $\uparrow$ operator is. It clearly doesn't bind the psychological state (Intention), which presumably holds at the time of the speech act in question; but it does bind the argument of the psychological state operator (indeed, if it is an operator). We will revisit this after gaining more experience of duality from a different, but closely related, perspective.

3 Present Tense, Some Aspect

In English, one usually can't use the simple present form to represent a simple (non-generic) event. For example, consider an English-speaking mountaineer talking with his mother by cell-phone as he reaches the summit:

Mountaineer I'm getting there, Mom, just as I told you I would. . . . But I'm a little short of breath. Not much oxygen up here.

Mother You can do it. And I only have so many minutes left this month, so hurry up and do it!

Mountaineer I'm reaching the top! / *I reach the top! / I've reached the top!

... ...

This is not a morphological oddity: there are some cases in which this general rule fails (so the issue is not about tense or aspect *per se*). Before we look at the cases where this generalization fails, let's try to understand why the generalization holds in the cases where it does.

3.1 The Baseline Theory

Suppose that to use the present tense in English to describe a state of affairs, the state of affairs has to hold at the time of utterance and beyond. This is true for sentences describing states. But it precludes the use of present tense sentences whose verbs describe events with a definite end point. Because: if we haven't reached the endpoint, the sentence isn't true; and if we've reached the endpoint, the event is already past. (We can't just coincide with the endpoint, because of the "beyond" clause of "at the time of utterance and beyond".) This Baseline Theory captures the judgements represented above.

Now, let's consider some potential counterexamples.

3.2 Narrative description

Here's a section from the diary of the mountaineer above:

> I'm very determined today. When the sun comes up, I'm already at the col. The ice-field is treacherous, but by mid-afternoon, I reach the top. (And Mom, on the phone, is completely supportive!)...

Narrative involves a temporal chain of events. Let's suppose that it can take one of two modes. In one mode (the descriptive mode), the events are taken to exist in time and the narrative attempts to describe them. In the other mode (the constructive mode), we start from scratch in some sense[5] and build up the event structure piece by piece in the narration. This is a case of duality—matching linguistic descriptions to events, whether real or imagined, is a game whose matching rules are the same, but whose interaction with time is reversed. In the descriptive mode, time and the temporal connections with other events (including the circumstances of the communicative event) are a given and these connections are respected. In the constructive mode, a temporal model is built from the description.

The Baseline Theory applies only to the descriptive mode, where it is perhaps useful to think that we measure temporal descriptions against an independently existing temporal flow of events. In the constructive mode, there is no independently existing temporal flow of events to compare against: we're construcing the temporal relations as we go.

4 Aspect in world-to-words cases

Now let us go back to Searle's classification and suppose that in the language games we play in conversation, we have some high level options, including the possibility of Searle's assertives and declarations, whose differentiation in Searle's taxonomy rests in part on the direction of fit parameter. Searle's characterization of direction of fit for declarations begins by contrasting declarations with other types:

> The illocutionary force indicating device in the sentence operates on the propositional content to indicate among other things the direction of fit between the propositional content and reality. In the case of assertives, the direction of fit is words-to-world, in the case of directives and commissives, it is world-to-words; in the case of expressives there is no direction of fit because the existence of fit is presupposed. The utterance can't get off the ground unless there is already a direction of fit. But now with the declarations we discover a very peculiar relation. The performance of a declaration brings about a fit by its very successful performance. How is such a thing possible? (p. 150)

Searle's proposed answer to this question is: "...the direction of fit is both words-to-world and world-to-words because of the peculiar character of declarations" (p. 150). He continues:

[5] We can never really start completely from scratch—it would be as if we could write before being born. But writers play with the notion of common ground in amazing ways: Kafka's great story 'The Bucketrider' is a case in point.

> The reason there has to be a relation of fit arrow here at all is that declarations do attempt to get language to match the world. But they do not attempt to do it either by describing an existing state of affairs (as do assertives) or by trying to get someone to bring about a future state of affairs (as do directives and commissives). (p. 150)

I prefer to think of the distinction as one based on dynamic model-theory: assertives are evaluated or tested against a (partial) model μ; successful declarations extend the model μ to a new (still partial) model μ'.

One reason for this preference is that it embeds the problem in a useful theoretical space.

A second (noted earlier) is that Searle's use of direction of fit indicators does not always clearly indicate how it is to be applied to the components of the linguistic object being modeled: in the classificatory schemata that Searle introduces and employs, the direction of fit indicator occupies the second position (after the symbol representing the illocutionary point), but the words relevant to the direction of fit are sometimes provided by the expression as a whole (as in many assertives), and sometimes by the complement (as in a commissive utterance of I promise to help you).

Third, the proposal that direction of fit goes both ways in declarations, while avoiding the initial dilemma posed, is less obvious than it appears at first glance. Are the two directions of fit independent? Are they temporally ordered? Do they apply equally to all components of the relevant structure? Thus, while the appeal to a two-way direction of fit for declarations is theoretically elegant, it isn't clear, to me at least, that it's correct.[6]

Finally, the model-theoretic perspective sheds light on the aspectual properties of declarations: unlike present tense sentences evaluated or tested against a given cotemporaneous model μ, the progressive form is not required. In fact, when the progressive form is used—compare *I name this ship the* U.S.S. Aliscafo and *I'm naming this ship the* U.S.S. Aliscafo—the special declarational force of the non-progressive vanishes and the sentence is understood as describing what the speaker is doing, rather than bringing about a name-entity pairing. Specifically, like the narrative constructive mode, declarations are exempt from the requirements of the Baseline Theory—because they are not evaluated against an existing, independent model. Of course, in virtue of duality, the verbs used in declarations can also be used, possibly with different tense and aspect to describe declarational events, as in the simple discourse: *I hereby name this ship the* U.S.S. Aliscafo. *There! I named the* Aliscafo *the* Aliscafo.

5 The Whiff of Generality

In fact, explicit performatives always have the aspectual properties just discussed with regard to declarations. The most direct conclusion to draw from this (in the framework adumbrated here, at least) is that explicit performatives, when succesitul, are always interpreted as model-extenders, rather than model-describers. This holds across all of Searle's classes when instances of them involve explicit performatives.

[6] I'm reminded for some reason of Russell's aside after introducing the problem of the baldness of the present king of France: "Hegelians, who love a synthesis, will probably conclude that he wears a wig." (Russell 1905:p. 48).

assertives: *I state that I am here.*
directives: *I request that you be there.*
commissives: *I promise to be there.*
expressives: *I thank you.*
declarations: *I name this dog* Lola.

From the present point of view, this means that at the top level, all of these sentences have a uniform characterization: as explicit performatives, they are all to be interpreted as model-extenders. How then do we characterize the distinctions among them? And what does this mean for Searle's taxonomy? We can't address these questions in full here. But a few comments are in order.

First, the parametric criteria proposed by Searle certainly remain relevant (especially the role of the direction of fit with regard to the complement clauses of the examples above). But notions like direction of fit need to be associated with the correct corresponding component of the expression in question. (That is, we would like a more compositional account.)

Second, the classification seems skewed: compare the narrowness and specificity of the commissives class (promises) to the almost open-ended character of the expressives class.

A natural question to ask, then, is whether (even given Searle's extensive set of parameters) the space of possibilities is appropriately chosen. For example, a promise involves not only an obligation on the part of the promiser but also a benefit to the promisee. Moreover, the promisee can act to unwind the promiser's obligation, for example by saying *I release you from your promise to X* or *You needn't worry about X-ing* or *Don't feel that you have to X... At least not on my behalf*. This type of act doesn't fit comfortably in any of Searle's classes. But such acts would be expected in a setting where speaker/hearer interactional negotiations of power/permission/obligation are a central and basic focus of communication.

Such a setting would also naturally accommodate the performative aspects of deontic uses of *may*. If we compare questions of the form *May I ... ?* with questions of the form *Am I allowed to ...*, we find that the former favor interpretations as requests for permission, not simply as requests for information, while the latter are more neutral. For example, if my brother Atreus is imprisoned for some reason and I wish to ascertain the boundaries of his freedom under these limited conditions, I might ask him "*Are you allowed to spend time outside your cell on weekdays?*", but the similar question "*May you spend time outside your cell on weekdays?*" is deontically odd, to say the least. If we assume that the first is a question about objective rules and regulations, while the second is a request for permission addressed to a person not in a position to grant permission, this oddity is immediately explained.

Third, many of the examples that Searle includes in his expressives category involve the interaction of the relation between speaker and hearer and events that affect the interests of speaker or hearer (congratulations, apologies, thanks, condolences, ...). Situating these speaker/hearer actions in an appropriate space should be useful.

But these questions, however interesting, go far beyond the bounds of duality. And addressing them must therefore be postponed to a more appropriate occasion.

6 Conclusion

Present tense sentences in English whose main verb is non-stative are subject to restrictions which are attributed above to a basic distinction: assertive uses of the sentence (where

the sentence describes an independently given model) and several different (narrative, performative) uses in which the sentence serves to create, in some way, the state-of-affairs which it describes. The Baseline Theory, described above in section §3.1 shows how this interesting range of grammatical behaviors can be characterized for basic cases. Duality provides the connection across these distinct uses. And we see that apparent counterexamples to the Baseline Theory—at least both the narrative and performative cases considered here—fall outside its domain of definition, for the same reason.

References

Anscombe, G. E. M. 1957. *Intention*. Ithaca, New York: Cornell University Press. Second Edition, 1969.

Austin, J. L. 1970. *How to do things with words*. New York: Oxford University Press. Paperback Edition.

Barr, Michael, and Charles Wells. 1990. *Category theory for computing science*. Prentice Hall International Series in Computer Science. New York: Prentice Hall.

Birkhoff, Garrett. 1967. *Lattice theory*, volume 25 of *Colloquium Publications*. Providence, R.I.: American Mathematical Society, third edition. Eighth Printing, with corrections, 1995.

Davey, B. A., and H. A. Priestley. 1990. *Introduction to lattices and order*. Cambridge Mathematical Textbooks. Cambridge: Cambridge University Press.

Halmos, Paul. 1960. *Naive set theory*. The University Series in Undergraduate Mathematics. New York: Van Nostrand Reinhold Company.

Kaplansky, Irving. 2003. *Linear algebra and geometry: A second course*. Mineola, New York: Dover. Originally published by Allyn & Bacon in 1969, and in a revised edition by Chelsa Publishing in 1974.

Keenan, Edward L., and Edward P. Stabler. 2003. *Bare grammar: Lectures on linguistics invariants*. Stanford Monographs in Linguistics. Stanford, California: CSLI Publications.

MacLane, Saunders. 1971. *Categories for the working mathematician*, volume 5 of *Graduate Texts in Mathematics*. Berlin: Springer.

Russell, Bertrand. 1905. On denoting. *Mind* new series, 14:479–493. Reprinted in (**?**:41–56).

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