UCLA Working Papers in Linguistics, Theories of Everything
Volume 17, Article 10: 69-71, 2012

Intensional quantifiers

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Introduction

In their classic paper on quantifiers Keenan and Stavi (1986) argue that determiners like *many* and *few* represent intensional relations between properties. In previous work using the rich type theory TTR (type theory with records), for example, Cooper (2011), I have introduced quantifiers as a relation between intensional properties but reduced this to the classical extensional relations between sets. This seems to be missing an opportunity offered by the intensional type theory. However, it seems that the nature of the intensionality in these examples is different from that to be found in attitude reports.

1 Keenan and Stavi's original examples

Keenan and Stavi (1986) discuss the examples in (1).

(1) a. Many lawyers attended the meeting this year
    b. Many doctors attended the meeting this year

In a situation where the set of lawyers attending the meeting this year is identical with the set of doctors attending the meeting this year it is still possible for the two sentences to get different truth values if for example 1000 doctors normally attend the meeting but only 500 show up this year making (1b) arguably false whereas only 20 lawyers normally attend the meeting and this year's showing of 500 is exceptionally many, making (1a) arguably true.

2 Treating quantifiers in TTR

TTR (type theory with records, Cooper 2012) is a *rich* type theory which adopts many ideas and techniques from Martin-Löf type theory, while at the same time using a more classical set-theoretic foundation familiar to traditional formal semantics. The "richness" lies in the fact that the type theory provides a greater variety of types than, for example, Montague's type theory. In addition to types for basic ontological classes such as individuals, sets and functions of various kinds, a rich type theory provides types corresponding to classifications of objects including, in the case of TTR, also situations. Fundamental to this kind of type theory is the notion of *judgement* that an object a is of type T, $a : T$. Types, in particular types of situations, may play the role of propositions, following the so-called "propositions as types" dictum. Truth corresponds to a type being non-empty (having a

witness), that is the set $\{a \mid a : T\}$ for a type T is not the empty set. Types are objects in their own right which are not defined in terms of their extension. That is, $\{a \mid a : T_1\} = \{a \mid a : T_2\}$ does not imply $T_1 = T_2$. Types can be constructed from other objects and may be structured objects with components. Predicates are a kind of type constructor. Thus if a and b are of type *Ind(ividual)*, hug(a,b) could be regarded as the type of situation in which a hugs b. It will be true that a hugs b just in case this type is non-empty.

Following this view, we can treat determiner relations, as predicates, q, which are type constructors which combine with two properties, P_1, P_2 to make a type, $q(P_1, P_2)$. Properties are treated basically as functions from individuals to types.[1] An individual *a has a property* P just in case $P(a)$ is a non-empty type. Thus the hyperintensionality of types is transmitted to properties: if it is the case that for any a, a has P_1 iff a has P_2 this does not imply that $P_1 = P_2$. However, in Cooper (2011) and other preceding work I have not exploited this intensionality but rather shown how this type-theoretic view of quantifiers can be related to classical extensional generalized quantifier theory. That is, a type $q(P_1, P_2)$ is required to be non-empty just in case the classical extensional quantifier relation corresponding to q holds between $\{a \mid a \text{ has } P_1\}$ and $\{a \mid a \text{ has } P_2\}$. This is obviously missing an opportunity for the analysis of intensional quantifier relations.

3 Is there just one kind of intensionality?

The claim we make for the kind of intensionality offered by this type theory is that it does not fall prey to the problems of the classical possible worlds approach in that two distinct types can be logically equivalent. It shares this with property theory and other theories which decouple propositions (here modelled as types) from sets of possible worlds. This yields a finer grain for intensional objects but there is always the question of whether the result is too fine grained. Our claim is that you are in a better position starting with an intensional system with too fine a grain rather than one with too coarse a grain. If the grain is too fine you can always look for equivalence classes which will correspond to what you need for semantic analysis. If you start from a system with too coarse a grain there is nothing you can do except add in additional structural objects to make the distinctions which are not there. (This is what has happened in various approaches to structured meanings.)

However, a problem seems to arise with fine grained intensionality when it comes to intensional quantifier relations. Consider a variant of example (1) where the noun arguments are logically equivalent.

(2) a. Many large prime numbers have been discovered by computer generation
    b. Many large numbers divisible only by themselves or one have been discovered by computer generation

It seems to me hard to imagine a situation where (2a) and (2b) have different truth values. At the very least it has a different feel to it than the doctors and lawyers example in (1). No such problems arise in a corresponding example involving an attitude verb.

(3) a. Sam believes that a large prime number has been discovered by computer generation

[1] Actually, in Cooper (2011) and elsewhere in TTR a property takes a record containing an individual to a record type, but the complication with records and record types is not relevant to the present discussion.

b. Sam believes that a large number divisible only by itself or one has been discovered by computer generation

It is easy to imagine a situation where (3a) and (3b) have different truth values. This causes me to speculate that there is not just one "intensional grain" at work in the language. One approach to making the distinction between the two kinds of intensionality could be based on the analysis of *many* and *few* in Lappin (2000) which involves crucial reference to normative situations which would not be used in the analysis of intensionality for the attitudes case. I would, however, prefer a commitment to normative situation types rather than normative situations. It seems like this should be related to work that Lappin and I (together with Simon Dobnik and Staffan Larsson) have in progress on probabilistic type theory.

Conclusion

The example we have seen here is one where the grain seems different for intensional quantifier relations and attitude relations. I wonder if in addition the individuation of intensional objects can depend on context and the knowledge resources available to agents. I would like to be able to argue that a system like type theory which introduces a basic very fine grain and then requires you to create equivalences to coarsen the grain is in a better position to deal with varying grains than a theory that commits you to a coarse grain from the beginning.

References

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